# MATHLETICS

# Chance and Probability

Name

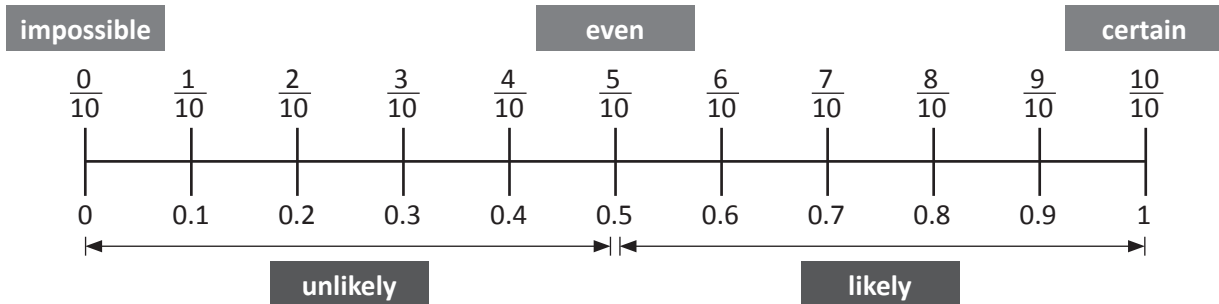# Series G – Chance and Probability

## Contents

Series Authors:

Rachel Flenley
Nicola Herringer

# Chance and probability – probability scale

Probability measures how likely something is to happen.

| impossible | | | | | even | | | | | certain |

$$\frac{0}{10} \quad \frac{1}{10} \quad \frac{2}{10} \quad \frac{3}{10} \quad \frac{4}{10} \quad \frac{5}{10} \quad \frac{6}{10} \quad \frac{7}{10} \quad \frac{8}{10} \quad \frac{9}{10} \quad \frac{10}{10}$$

0    0.1    0.2    0.3    0.4    0.5    0.6    0.7    0.8    0.9    1

**unlikely**                    **likely**

**1** Probability measures how likely something is to happen. Events that are certain to happen are given a probability of 1. Events that will never happen are given a probability of 0. Events that could happen are rated between 0 and 1.

| Event | Probability as a fraction | Probability as a decimal |
|---|---|---|
| When you flip a coin, it will land on heads. | | |
| You will grow wings and fly today. | | |
| A spinner with 10 even segments with the numbers 1 to 10 will land on 3. | | |
| 5 people are lined up and every second person in the line has gloves on. What is the chance that one person is not wearing gloves? | | |
| You have 20 cards. 5 have hearts, 5 have stripes and the rest are blank. What is the chance you will choose a blank card? | | |

**2** What is the probability of spinning a striped segment on each of these wheels? Write your answer as a rating between 0 and 1 using decimals.

a              b              c              d
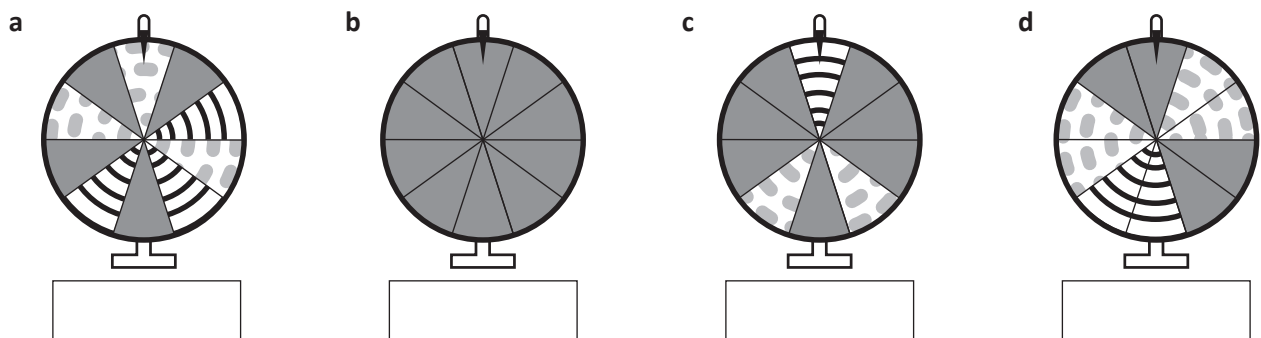
**3** Reuben is going to put ten blocks in a bag and ask a friend to choose one without looking. Circle the blocks he could put in the bag to make the probability of choosing a cube $\frac{2}{10}$.
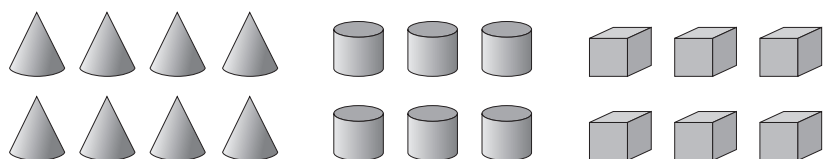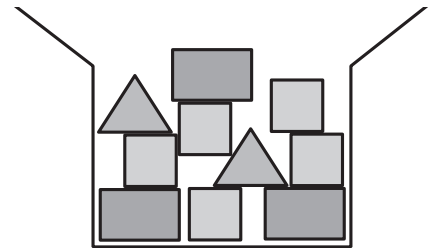
# Chance and probability – probability scale

**4** **100 guests each buy a ticket for a raffle at a fundraising dinner. The winning ticket will be selected at random. This table on the right shows the colours of all of the tickets in the raffle.**

| Red | 10 |
|---|---|
| Purple | 40 |
| Orange | 50 |
| **Total** | **100** |

What is the probability of the winning ticket being red, purple or orange? Draw arrows on this probability scale to show the probability of each colour and write the colour beneath the arrow.

```
0    0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9    1
├─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┤
```

................................................................................................

**5** **Inside a box there are 3 rectangles, 2 triangles and 5 squares. Without looking, Ellie chooses one shape from the box.**

**a** Draw each shape on this probability scale to show the probability of Ellie choosing each type of shape.

```
0    0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9    1
├─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┤
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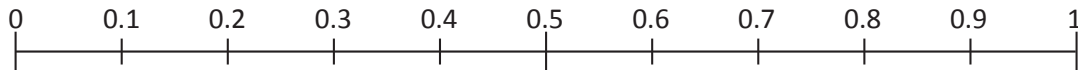
**b** 3 more rectangles, 2 more triangles and 5 more squares are added to the same box. Draw each shape on this probability scale to show the probability of Ellie choosing each shape from the box.

```
0    0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9    1
├─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┼─────┤
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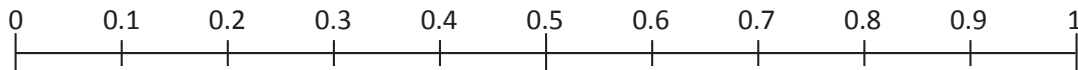
**c** What do you notice? _____

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**6** **Sam did an experiment with 10 cubes that were either red, white or blue. She took a cube from a jar without looking, tallied which colour it was then put it back in the same jar. She repeated the process 20 times. After tallying her results, she created this pie chart to show the results of the experiment.**

**a** How many times did Sam take each colour out of the jar? Remember she performed the experiment 20 times.
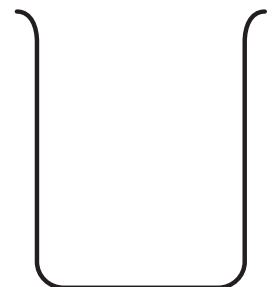
Red ☐

White ☐

Blue ☐
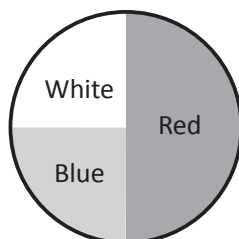
**b** Draw the combination of cubes there could have been inside the jar. Remember there are only 10 cubes.

G 1

SERIES   TOPIC

**Chance and Probability**

# Chance and probability – using samples to predict probability

Surveys are used to collect data about certain topics or questions. Once the data is collected, it is presented in a table so it is easy to understand. Surveys can be conducted to ask all kinds of questions.

We can use probability to see an even bigger picture than the survey tells us.
This table shows the data collected when 50 people were surveyed to find their favourite milkshake flavour.

| Chocolate | Strawberry | Vanilla | Banana |
|-----------|-----------|---------|--------|
| 19 | 16 | 8 | 7 |

We can use probability to predict the number of people who will choose each flavour in a larger survey. When 100 people are surveyed, it is likely that chocolate will be the favourite milkshake flavour of 38 people.
When 1 000 people are surveyed, it is likely that chocolate will be the favourite milkshake flavour of 380 people.

1  **Faisal has had enough of selling clothes. If one more woman asks him, "Do I look fat in this?", he will scream. He holds a crazy closing down sale and sells the following items in 1 hour:**

| Shirts | Jackets | Skirts | Dresses |
|--------|---------|--------|---------|
| 18 | 14 | 7 | 3 |

**Predict how many:**

a   jackets would sell in 2 hours

b   skirts would sell in 2 hours

c   shirts would sell in 3 hours

d   dresses would sell in 4 hours

e   shirts and jackets would sell in 4 hours

f   items of clothing would sell in 8 hours

2  **Here is a table showing the results from a survey of 50 boys and 50 girls who were asked, "Which fruit do you like best?" Rate the probability that a person selected randomly will be:**

a   a boy

b   a girl who likes apples

|        | Girls | Boys |
|--------|-------|------|
| Apple  | 17 | 11 |
| Banana | 8 | 14 |
| Orange | 13 | 16 |
| Pear   | 12 | 9 |

c   someone who likes pears

d   Is the probability of someone choosing a banana greater than or less than $\frac{1}{2}$? _____

# Chance and probability – tree diagrams

Tree diagrams are used to display all possible outcomes in a simple chance experiment. Here is an example: Matilda's father is making her lunch and has given her the following choice: white or brown bread, lettuce or cucumber, tuna or egg. We can then follow each branch along to see the different options.

sandwich

white bread
- lettuce — tuna / egg
- cucumber — tuna / egg

brown bread
- lettuce — tuna / egg
- cucumber — tuna / egg
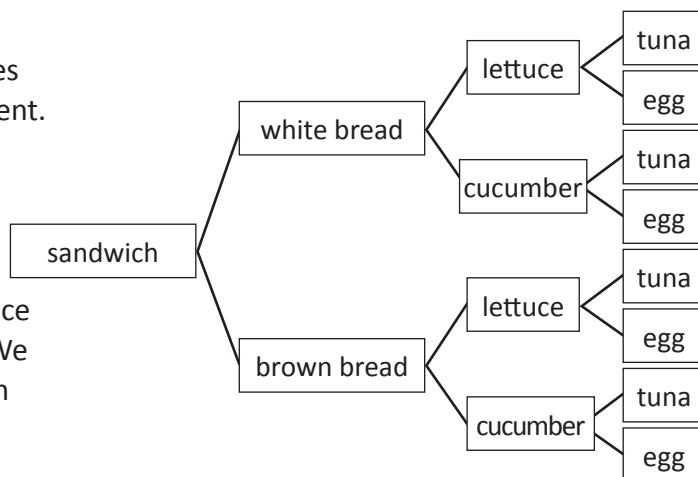
By using a tree diagram, we can see that Matilda has 8 different options for her sandwich.

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**1** When customers buy a new car from Joe's Motors they can pay an additional cost for each of these optional extras:
- Alloy wheels instead of standard wheels
- Automatic transmission instead of manual transmission
- Metallic paint instead of standard paint
- Leather seats instead of standard seats

**a** Complete this tree diagram to work out all the possible combinations that customers can choose:

Automatic transmission

Manual transmission
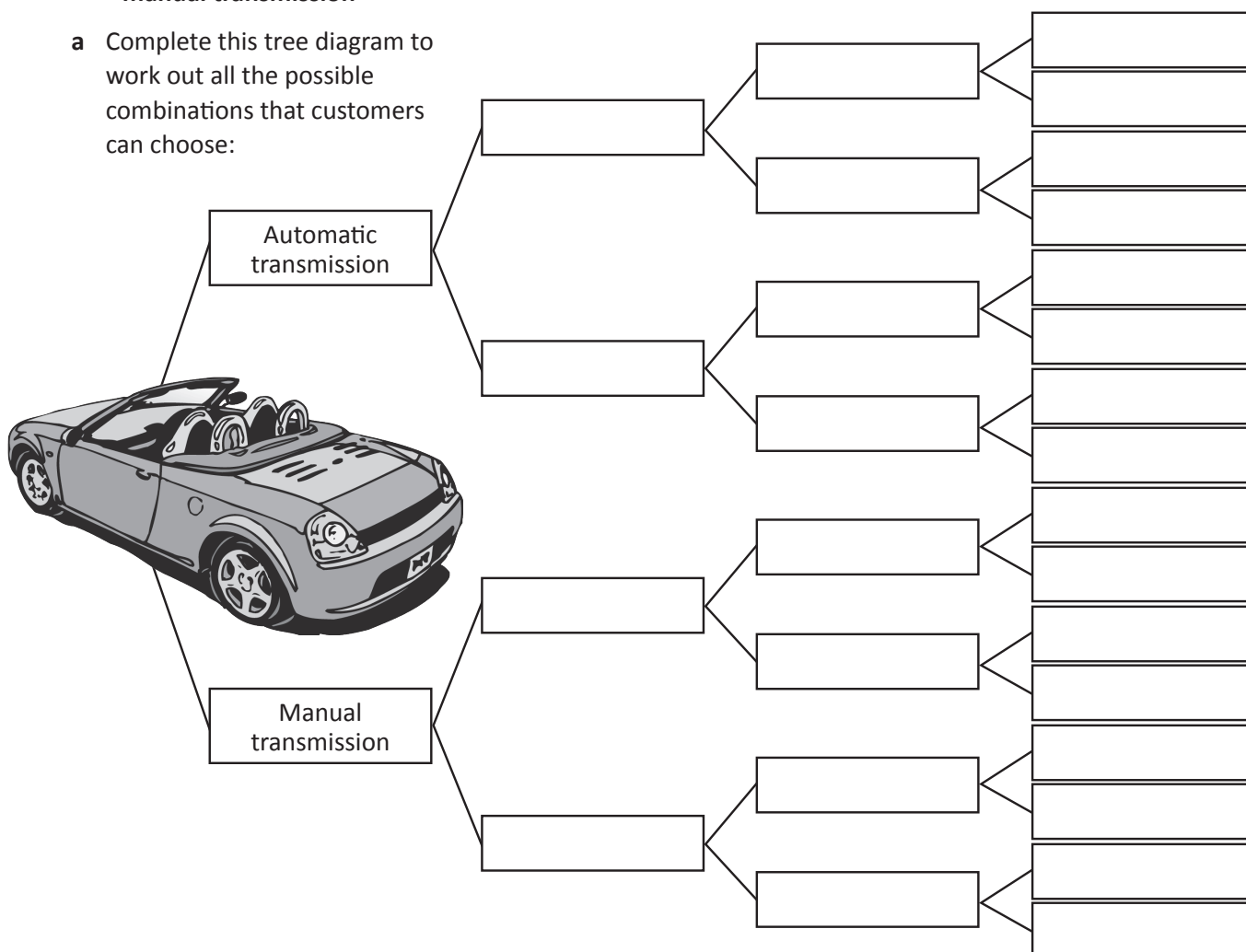
**b** How many possible combinations are there? _____

SERIES G1 TOPIC

**Chance and Probability**

# Chance and probability – tree diagrams

**2**  You have an after school job at the local ice-cream shop. Your boss has asked you to run a special on the strawberry and banana ice-cream flavours as she mistakenly ordered far too much of each.

You decide to offer a double scoop special – customers can choose 2 scoops and a topping for the price of a single scoop cone. As all ice-cream connoisseurs know, it matters which flavour goes on top so customers may choose a strawberry-banana combo, a banana-strawberry combo or 2 scoops of the same flavour.

Work out the different combinations customers could order if they could choose from 2 cone types, the 2 flavours and 2 different toppings. Decide which cones and toppings you will offer.
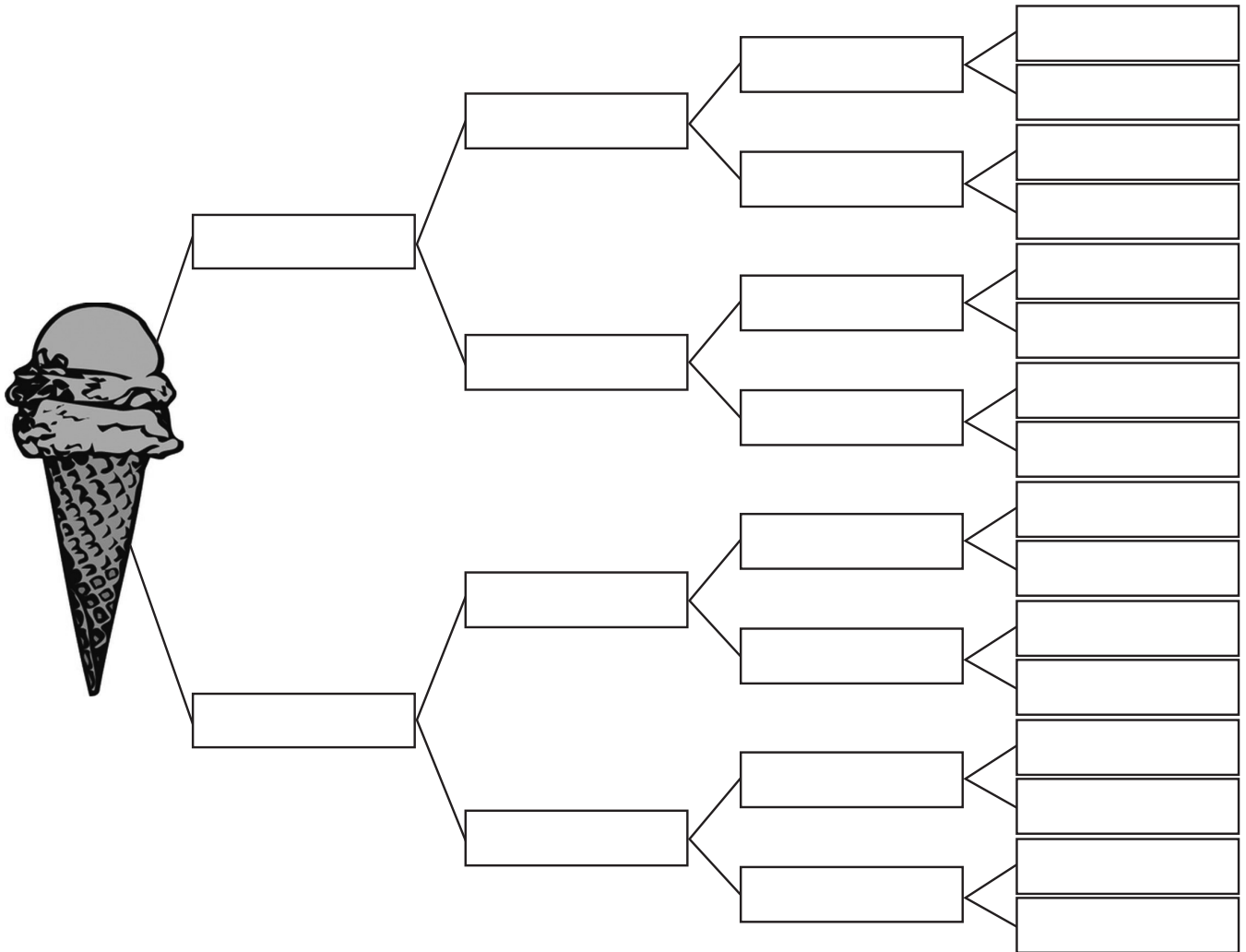
**3**  **Think about this:**

**a**  How many different combinations are there in total?

**b**  If a customer hates banana ice-cream flavour, how many options do they have?

**c**  What would be your pick?

# Chance and probability – chance experiments

**1** Complete the tree diagram to show all the possible outcomes when you spin Spinner 1 and then Spinner 2. The first one is done for you.

| blue | → | 1 |
| | | 2 |
| | | 3 |
| | | 4 |
| | | 5 |

**blue**

**yellow**

**red**

**Spinner 1**

| 1 | 2 |
| 5 | | 3 |
| | 4 | |
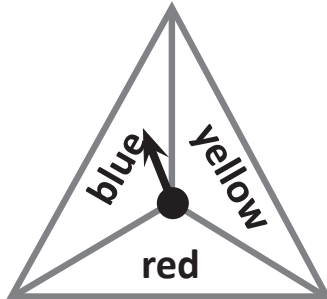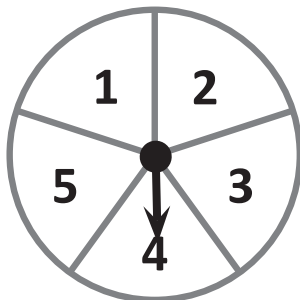
**Spinner 2**
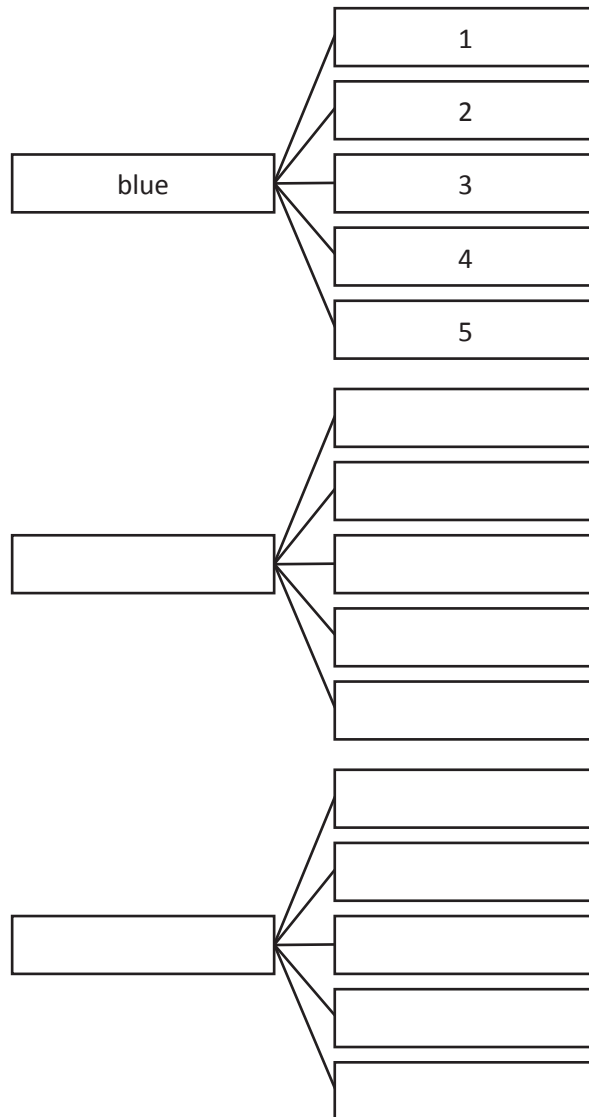
**2** What is the probability of landing on:

**a** a yellow

**b** blue and 1

**c** a 4

**d** yellow and 3

> There were 15 possible outcomes in Question 1. 60 is 4 × 15, so I would expect each number to be 4 times greater.

**3** If you did this 60 times, how many times would you expect to get:

**a** blue and 4

**b** a red

**c** a 1

**d** a 5

*THINK*

**Chance and Probability**

# Chance and probability – using tables

When we work out all the possible outcomes of an event that could happen, we are finding out the theoretical probability. When we do the experiment and look at the probability of what actually happened, we call it experimental probability.

Theoretical probability is:

$$\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

Experimental probability is:

$$\frac{\text{number of times the event occurred}}{\text{total number of trials}}$$

**1** When we roll 2 dice together, we can get a number of totals. Fill in this table to show the possible outcomes when 2 regular dice are rolled and added together:

| | Die 1 | | | | | |
|---|---|---|---|---|---|---|
| **+** | **1** | **2** | **3** | **4** | **5** | **6** |
| **1** | | | | | | |
| **2** | | | | | | |
| **3** | | | | | | |
| **4** | | | | | | |
| **5** | | | | | | |
| **6** | | | | | | |

(Die 2 labels the rows)

**a** How many different ways can the dice be rolled?

**b** Which total occurred the most often? Shade this in the grid.

**c** Which totals occurred the least often? Circle these on the grid.

**2** Graph the outcomes from the table above in the grid below.
Express the theoretical probability of the following as a fraction:

Number of outcomes

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Possible totals

**a** 7 = ⬚/⬚    **b** 9 = ⬚/⬚

**c** 2 = ⬚/⬚    **d** 10 = ⬚/⬚

**3** Now try this experiment. You will work with a partner and roll 2 dice 36 times. First make your predictions as to how often you will roll each answer. Write this in the first row. This is the _____ probability. Now actually roll two dice 36 times. In the bottom row, tally the number of times each total appears. This is the _____ probability.

| Total | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of times you expect to see each total | | | | | | | | | | | |
| Number of times you actually get each total | | | | | | | | | | | |

**4** Look at the difference between the two rows. Is this what you expected?

# Chance and probability – using tables

Now we are going to investigate the sample space of when the dice are different to regular dice. For this you will need 2 regular dice and some white stickers to stick over the sides of the dice.

**5** Cover 2 dice with white stickers so that the sides are covered on each die. Colour 4 of the faces yellow and colour 2 faces red:
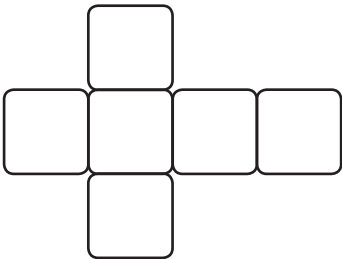
|  |  | Die 1 |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
|  | **+** | **Y** | **Y** | **Y** | **Y** | **R** | **R** |
|  | **Y** | YY |  |  |  |  |  |
|  | **Y** |  |  |  |  |  |  |
| **Die 2** | **Y** |  |  |  | YR |  |  |
|  | **Y** |  |  |  |  |  |  |
|  | **R** | RY |  |  |  |  |  |
|  | **R** |  |  |  |  |  |  |

**a** Complete the table to show the sample space.

**b** What are the chances of rolling 2 yellows? Colour the table to show this.

**c** What are the chances of rolling 1 yellow and 1 red?

**d** What are the chances of rolling 2 reds?

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**6** Look at the next table for the sample space of a set of dice.

|  |  | Die 1 |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
|  | **+** | **Y** | **Y** | **G** | **G** | **●** | **●** |
|  | **Y** | YY | YY | YG | YG | Y● | Y● |
|  | **Y** | YY | YY | YG | YG | Y● | Y● |
| **Die 2** | **G** | GY | GY | GG | GG | G● | Y● |
|  | **G** | GY | GY | GG | GG | Y● | Y● |
|  | **●** |  |  |  |  |  |  |
|  | **●** |  |  |  |  |  |  |

**a** Complete the rest of the table to show the sample space.

**b** Show what one die looks like on this net of a cube.

**c** What is the chance of rolling:

2 yellows?

2 dots?

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**7** Make up your own crazy set of dice. Show the sample in the space on the left and show what they look like on the two nets of cubes on the right.

|  |  | Die 1 |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
|  | **+** |  |  |  |  |  |  |
| **Die 2** |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

**Die 1**

**Die 2**

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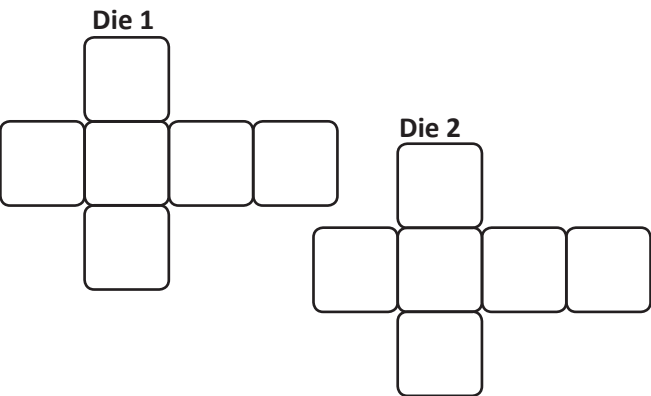
**Getting ready**

Play this game with a friend. You will need one copy of this game board, a counter each and two dice.
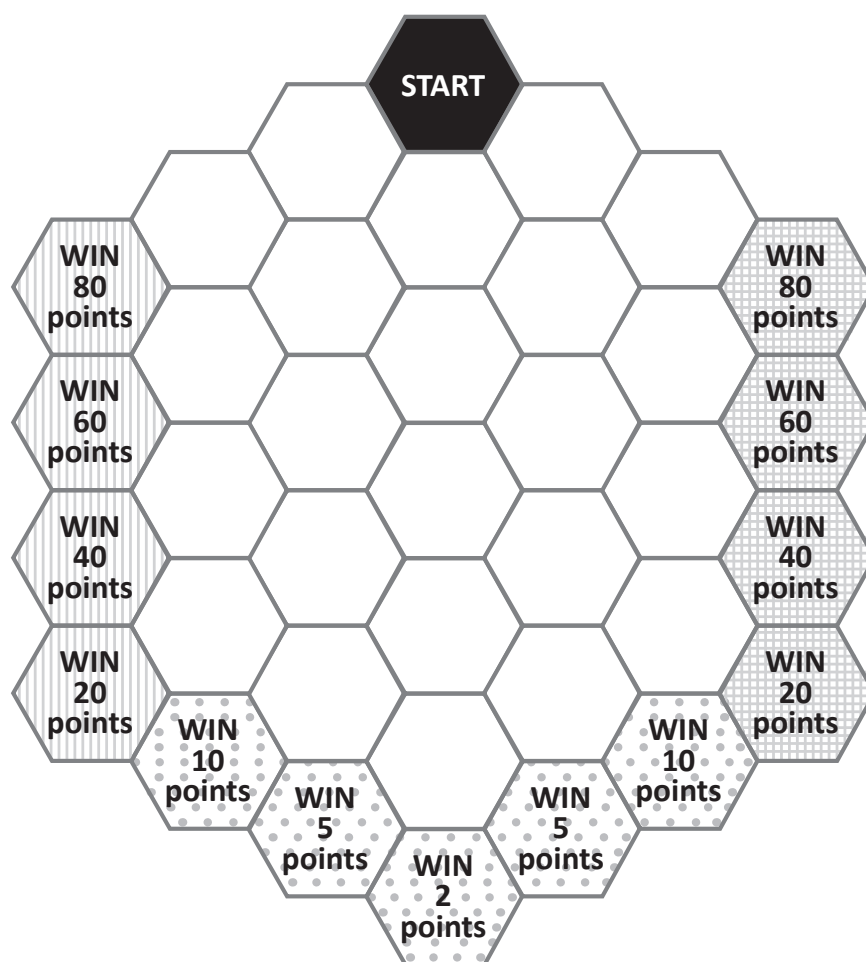
**What to do**

Place your counter in the start hexagon. Take turns rolling both dice and adding the numbers.

- If your answer is a 2, 3 or 4 move one space towards the striped hexagons.
- If your answer is a 5, 6, 7, 8 or 9 move one space towards the dotted hexagons.
- If your answer is a 10, 11 or 12 move one space towards the checked hexagons.

When your counter gets to a hexagon on the edge, record your points and start again.

Play 5 games. Who is the grand winner?

**START**

WIN 80 points | WIN 80 points

WIN 60 points | WIN 60 points

WIN 40 points | WIN 40 points

WIN 20 points | WIN 20 points

WIN 10 points | WIN 10 points

WIN 5 points | WIN 5 points

WIN 2 points

**What to do next**

Why are the points allocated as they are? Why does it matter what your dice roll is? Explain your reasoning to a friend.
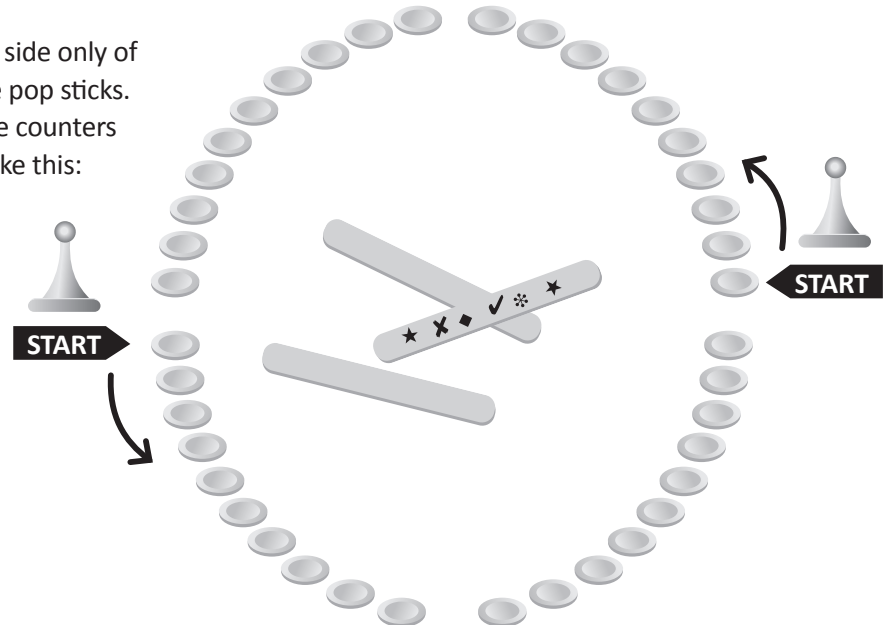
**Getting ready**

This is a version of a very old game, played by children all over the world. You will need 40 counters, 2 playing pieces (you could use erasers or chess pieces) 3 pop sticks and a partner.

**What to do**

Decorate 1 side only of each of the pop sticks. Arrange the counters in a circle like this:



Place your playing pieces on opposite sides of the circle and mark your starting point. The aim of the game is to be the first person to move around the circle and get back to your starting point.

Take turns throwing the 3 pop sticks up and looking at the result. The number of counters you can move depends on your combination of decorated and undecorated pop sticks:

- 3 decorated sides = move 10 counters
- 3 plain sides = move 5 counters
- 2 decorated sides and 1 plain side = move 3 counters
- 1 decorated side and 2 plain sides = move 1 counter

If the other player lands on you, you must return to your starting point. The first person back to the Start wins.

**What to do next**

After you finish the game, make a tree diagram of all the possible throw outcomes. Use the diagram to answer the following questions:

- What is the likelihood of throwing 3 decorated sides?
- What is the likelihood of throwing 3 plain sides?
- What is the likelihood of throwing 2 decorated and 1 plain sides?
- What is the likelihood of throwing 1 decorated and 2 plain sides?

Based on this, do you think the scoring system is fair? How would you change the scoring system to make it fairer? Play the game again with your new scoring system. Does this improve the game? Or do you prefer the original game? Why?

**Chance and Probability**